



HEAT TRANSFER ANALYSIS OF STRAIGHT FINS WITH VARIABLE CONDUCTIVITY USING ADOMIAN DECOMPOSITION METHOD

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(Geliş Tarihi: 16. 07. 2004)

Abstract: The Adomian decomposition method (ADM) has been used to determine the temperature distribution within straight fins with convective tip and temperature-dependent thermal conductivity and to evaluate the temperature gradient at the fin tip. The method is useful and practical for solving the non-linear heat diffusion equation which is associated with variable thermal conductivity condition. The ADM provides an analytical solution in the form of an infinite power series. The fin temperature distribution has been obtained as a function of Biot number at the fin tip, thermo-geometric fin parameter and thermal conductivity parameter describing the variation of the thermal conductivity. The present results can be used for designing of straight fins with temperature-dependent thermal conductivity and convective tip.

Keywords: Adomian decomposition method, Straight fin, Variable conductivity

DEĞİŞKEN ISI İLETİM KATSAYILI DÜZ KANATLARIN ADOMIAN AYRIŞTIRMA YÖNTEMİ İLE ISIL ANALİZİ

Özet: Bu çalışmada, ısı iletim katsayısı değişken olan ve ucundan taşınım ile ısı kaybeden düz bir kanat içerisindeki sıcaklık dağılımını bulmak ve kanat taban kesitindeki sıcaklık gradyanını elde etmek için Adomian ayrıştırma yöntemi kullanılmıştır. Sonsuz kuvvet serisi şeklinde bir analitik çözüm sağlayan Adomian ayrıştırma yöntemi, ısı iletim katsayısının değişken olması yüzünden doğrusal olmayan ısı yayılım denklemini çözmek için kullanışlı ve pratik bir yöntemdir. Kanat içindeki sıcaklık dağılımı; kanat ucundaki Biot sayısı, termo-geometrik kanat parametresi ve ısı iletim katsayısının değişimini tanımlayan ısı iletim parametresinin fonksiyonu olarak elde edilmiştir. Bu çalışmada elde edilen sonuçlar ısı iletim katsayısı sıcaklıkla değişen ve ucundan taşınım ile ısı geçişi olan düz kanatların tasarımında kullanılabilir.

Anahtar Kelimeler: Adomian ayrıştırma yöntemi, Düz kanatlar, Değişken ısı iletim katsayısı

NOMENCLATURE

A_c	fin cross-sectional area (m^2)
A_m	Adomian polynomials
b	fin length (m)
Bi	Biot number at the fin tip
C	an integral constant arising in Eq. (20)
h	heat transfer coefficient [$W/(m^2K)$]
k	thermal conductivity [$W/(mK)$]
L	the higher order derivative
L^{-1}	inverse operator of L
M	thermo-geometric fin parameter
N	nonlinear terms
P	fin perimeter (m)
Q	total heat transfer rate (W)
q	dimensionless heat-transfer rate
R	remainder of the linear operator
T	temperature (K)
x	distance measured from the fin tip (m)

Greek letters

β	dimensionless parameter describing variation of the thermal conductivity
ξ	dimensionless coordinate
λ	the slope of the thermal conductivity-temperature curve (K^{-1})
θ	dimensionless temperature

Subscripts and superscripts

a	ambient environment
b	fin base
s	fin surfaces
t	fin tip
$'$	first derivative respect to ξ

INTRODUCTION

Extended surfaces are extensively used in various industrial applications. An extensive review on this topic is presented by Kern and Kraus (1972). Fins are employed to enhance the heat transfer between the primary surface and its convective, radiating or convective-radiating

environment. A considerable amount of research has been conducted into the variable thermal parameters which are associated with fins operating in practical situations. Aziz and Hug (1975) and Krane (1976) used the regular perturbation method and a numerical solution method to compute a closed form solution for a straight convecting fin with temperature dependent thermal conductivity. Razelos and Imre (1980) considered the variation of the convective heat transfer coefficient from the base of a convecting fin to its tip. A method of temperature-correlated profiles is used to obtain the solution of optimum convective fin when the thermal conductivity and heat transfer coefficient are functions of temperature (Sohrabpour and Razani, 1993). Laor and Kalman (1996) examined straight, spine and annular fins governed by power law-type temperature dependence of the heat transfer coefficient. Yu and Chen (1998) investigated the optimal fin length of a convective-radiative straight fin with rectangular profile under convective boundary conditions and variable thermal conductivity. Yu and Chen (1999) assumed that the linear variation of the thermal conductivity and exponential function with the distance of the heat transfer coefficient and then, solved the nonlinear conducting-convecting-radiating heat transfer equation by the differential transformation method. Bouaziz et al. (2001) presented the efficiency of longitudinal fins with temperature-dependent thermo-physical properties.

ADM is used for solving various nonlinear heat transfer problems (Vadasz and Olek, 1998; Lesnic and Elliot, 1999; Lesnic, 2002). Chiu and Chen (2002a) investigated a convective longitudinal fin with insulated tip and temperature-dependent thermal conductivity for optimal geometry of the fin. In another study carried out by Chiu and Chen (2002b), ADM is used for expressing the thermal stress within an annular fin.

In the present paper, a straight fin with convective tip and temperature-dependent thermal conductivity has been considered. The energy balance for a differential fin element is developed. The resulting nonlinear differential equation is solved by ADM to evaluate the temperature distribution within the fin. Using the temperature distribution, the temperature gradient at the fin base is expressed as a function of thermogeometric fin parameter, M , thermal conductivity parameter, β , describing the variation of the thermal conductivity and Biot number at the fin tip.

PROBLEM DESCRIPTION

Consider a straight fin with convective tip, arbitrary constant cross sectional area A_c , perimeter P , and length b . Fig.1 illustrates such a convective straight fin, and the following assumptions are made for the analysis.

1. Steady-state heat flow.
2. One-dimensional thermal analysis is considered.

3. There are no heat sources in the fin itself.
4. The thermal conductivity of the fin is a linear function of temperature.
5. The heat transfer coefficients over the fin surfaces and tip are constant but unequal.
6. The temperature of the surrounding fluid, T_a , is constant.
7. The temperature of the base of the fin, T_b , is uniform.

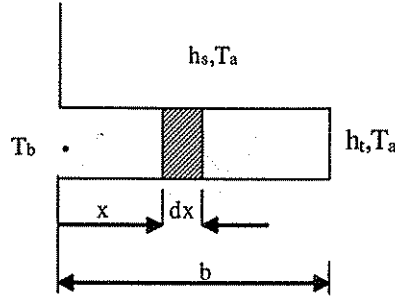


Figure1. Schematic diagram of a straight fin.

The one-dimensional energy balance equation is given

$$A_c \frac{d}{dx} \left[k(T) \frac{dT}{dx} \right] - Ph_s(T - T_a) = 0 \quad (1)$$

The thermal conductivity of the fin material is assumed to be a linear function of temperature according to

$$k(T) = k_a [1 + \lambda(T - T_a)] \quad (2)$$

where k_a is the thermal conductivity at the ambient fluid temperature of the fin and λ is the parameter describing the variation of the thermal conductivity.

Employing the following dimensionless parameters

$$\begin{aligned} \theta &= \frac{T - T_a}{T_b - T_a} & \xi &= \frac{x}{b} & \beta &= \lambda(T_b - T_a) \\ M &= \frac{Ph_s b^2}{k_a A_c} & Bi &= \frac{h_t b}{k_a} \end{aligned} \quad (3)$$

and assuming the thermal conductivity to be constant at the fin tip (since the tip temperature is approximate to the ambient fluid temperature, it can be assumed that the thermal conductivity at the fin tip is constant), the formulation of the problem reduces to

$$\frac{d^2 \theta}{d\xi^2} + \beta \theta \frac{d^2 \theta}{d\xi^2} + \beta \left(\frac{d\theta}{d\xi} \right)^2 - M\theta = 0 \quad (4a)$$

$$\theta = 1 \quad \text{at} \quad \xi = 0 \quad (4b)$$

$$\frac{d\theta}{d\xi} + Bi\theta = 0 \quad \text{at} \quad \xi = 1 \quad (4c)$$

THE ADOMIAN DECOMPOSITION METHOD

The Adomian decomposition scheme is a method for solving a wide range of problems whose mathematical models yield equation or system of equations involving algebraic, differential, integral and integro-differential (Adomian, 1988a; Adomian, 1988b; Adomian, 1994).

We consider a general nonlinear equation

$$Lu + Ru + Nu = g, \quad (5)$$

where L is highest order derivative which is assumed to be easily invertible, R the linear differential operator of less order than L , Nu represents the nonlinear terms, and g is the source term. Applying the inverse operator L^{-1} to the both sides of Equation (5), and using the given conditions we obtain

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu), \quad (6)$$

where the function $f(x)$ represents the terms arising from integrating the source term $g(x)$, and from the using given conditions, all of which are assumed to be prescribed.

For nonlinear differential equations, the nonlinear operator $Nu = F(u)$ is represented by an infinite series of the so-called Adomian polynomials

$$F(u) = \sum_{m=0}^{\infty} A_m \quad (7)$$

The polynomials A_m are generated for all kind of nonlinearity so that A_0 depends only on u_0 , A_1 depends on u_0 and u_1 , and so on.

The Adomian method defines the solution $u(x)$ by the series

$$u = \sum_{m=0}^{\infty} u_m \quad (8)$$

In the case of $F(u)$ the infinite series is a Taylor expansion about u_0 . In other words

$$F(u) = F(u_0) + F'(u_0)(u - u_0) + F''(u_0) \frac{(u - u_0)^2}{2!} + F'''(u_0) \frac{(u - u_0)^3}{3!} + \dots \quad (9)$$

By rewritten Equation (8) as $u - u_0 = u_1 + u_2 + u_3 + \dots$, substituting it into Equation (9) and then equating two expressions for $F(u)$ found in Equation (9) and Equation (7) defines formulas for the Adomian polynomials

$$F(u) = A_1 + A_2 + \dots = F(u_0) + F'(u_0)(u_1 + u_2 + \dots) + F''(u_0) \frac{(u_1 + u_2 + \dots)^2}{2!} + \dots \quad (10)$$

By equating terms in Equation (10) the first few Adomian's polynomials A_0, A_1, A_2, A_3, A_4 are given

$$A_0 = F(u_0), \quad (11a)$$

$$A_1 = u_1 F'(u_0), \quad (11b)$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!} u_1^2 F''(u_0), \quad (11c)$$

$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!} u_1^3 F'''(u_0) \quad (11d)$$

Now that the A_k are known, Equation (7) can be substituted in Equation (6) to specify the terms in the expansion for the solution of Equation (8).

FIN TEMPERATURE DISTRIBUTION

Following the Adomian decomposition analysis, the linear operator is defined as:

$$L = \frac{d^2}{d\xi^2}. \quad (12)$$

Consequently, Equation (4a) can be written as follows:

$$L\theta = -\beta\theta \frac{d^2\theta}{d\xi^2} - \beta \left(\frac{d\theta}{d\xi} \right)^2 + M\theta \quad (13)$$

$$= -\beta NA - \beta NB + M\theta$$

where

$$NA = \theta \frac{d^2\theta}{d\xi^2} = \sum_{m=0}^{\infty} A_m \quad (14a)$$

$$NB = \left(\frac{d\theta}{d\xi} \right)^2 = \sum_{m=0}^{\infty} B_m \quad (14b)$$

are nonlinear terms. Hence, using Equation (11) gives

$$A_0 = \theta_0 \frac{d^2\theta_0}{d\xi^2} \quad (15a)$$

$$A_1 = \theta_1 \frac{d^2\theta_0}{d\xi^2} + \theta_0 \frac{d^2\theta_1}{d\xi^2} \quad (15b)$$

$$A_2 = \theta_2 \frac{d^2\theta_0}{d\xi^2} + \theta_1 \frac{d^2\theta_1}{d\xi^2} + \theta_0 \frac{d^2\theta_2}{d\xi^2} \quad (15c)$$

$$A_3 = \theta_3 \frac{d^2\theta_0}{d\xi^2} + \theta_2 \frac{d^2\theta_1}{d\xi^2} + \theta_1 \frac{d^2\theta_2}{d\xi^2} + \theta_0 \frac{d^2\theta_3}{d\xi^2} \quad (15d)$$

and

$$B_0 = \left(\frac{d\theta_0}{d\xi} \right)^2 \quad (16a)$$

$$B_1 = 2 \frac{d\theta_0}{d\xi} \frac{d\theta_1}{d\xi} \quad (16b)$$

$$B_2 = \left(\frac{d\theta_1}{d\xi} \right)^2 + 2 \frac{d\theta_0}{d\xi} \frac{d\theta_2}{d\xi} \quad (16c)$$

$$B_3 = 2 \frac{d\theta_1}{d\xi} \frac{d\theta_2}{d\xi} + 2 \frac{d\theta_0}{d\xi} \frac{d\theta_3}{d\xi} \quad (16d)$$

Applying the inverse operator L^{-1} to both sides of Equation (13) we obtain

$$L^{-1}L\theta = -\beta L^{-1}NA - \beta L^{-1}NB + ML^{-1}\theta, \quad (17)$$

and

$$\theta = \theta_0 - \beta L^{-1}NA - \beta L^{-1}NB + ML^{-1}\theta. \quad (18)$$

If L is a second-order operator, L^{-1} is a twofold indefinite integral, i.e.

$$L^{-1}L\theta = \theta - \theta(0) - \xi \frac{d\theta(0)}{d\xi}. \quad (19)$$

thus,

$$\theta_0 = \theta(0) + \xi \frac{d\theta(0)}{d\xi}. \quad (20)$$

Employing the boundary condition given in Equation (4b), and attaining $d\theta(0)/d\xi$ to any arbitrary constant, C , Equation (20) is rewritten as follows:

$$\theta_0 = 1 + \xi C. \quad (21)$$

The next iterates are determined recursively by

$$\theta_{m+1} = -\beta L^{-1}A_m - \beta L^{-1}B_m + ML^{-1}\theta_m \quad (22)$$

Therefore, the first four iterates are expressed as:

$$\theta_0 = 1 + C\xi \quad (23a)$$

$$\theta_1 = \frac{1}{2}(M - \beta C^2)\xi^2 + \frac{1}{6}MC\xi^3 \quad (23b)$$

$$\begin{aligned} \theta_2 = \frac{1}{2}(\beta^2 C^2 - \beta M)\xi^2 + \frac{1}{6}(3C^3\beta^2 - 4\beta MC)\xi^3 \\ + \frac{1}{24}(M^2 - 5\beta MC^2)\xi^4 + \frac{1}{120}M^2C\xi^5 \end{aligned} \quad (23c)$$

$$\begin{aligned} \theta_3 = \frac{1}{2}(\beta^2 M - \beta^3 C^2)\xi^2 + \frac{1}{6}(7\beta^2 MC - 6\beta^3 C^3)\xi^3 \\ + \frac{1}{24}(28\beta^2 C^2 M - 15\beta^3 C^4 - 5\beta M^2)\xi^4 \\ + \frac{1}{60}(19\beta^2 C^3 M - 10\beta CM^2)\xi^5 \\ + \frac{1}{720}(M^3 - 21\beta C^2 M^2)\xi^6 + \frac{1}{5040}M^3 C\xi^7 \end{aligned} \quad (23d)$$

:

Upon summing those iterations, the n -term approximation is expressed as

$$\Phi_{n+1} = \sum_{m=0}^n \theta_m = \theta_0 + \theta_1 + \theta_2 + \dots + \theta_n \quad (24)$$

The sum Φ_{n+1} may serve as a practical solution and converges very rapidly.

HEAT TRANSFER RATE

Total heat transfer rate is expressed by applying the Fourier's law at the fin base, as

$$Q = -k_b A_c \left. \frac{dT}{dx} \right|_{x=0} \quad (25)$$

The dimensionless heat transfer rate is defined as follows:

$$q = \frac{Q}{k_a A_c (T_b - T_a)/b} = -(1 + \beta) \left. \frac{d\theta}{d\xi} \right|_{\xi=0} \quad (26)$$

Since the temperature gradient at the fin base equal to the integral constant C as mentioned above, the dimensionless heat transfer rate rewritten as

$$q = -(1 + \beta)C \quad (27)$$

The coefficient C can be evaluated from the boundary condition given in Equation (4c) using the Newton-Raphson method.

RESULTS AND DISCUSSION

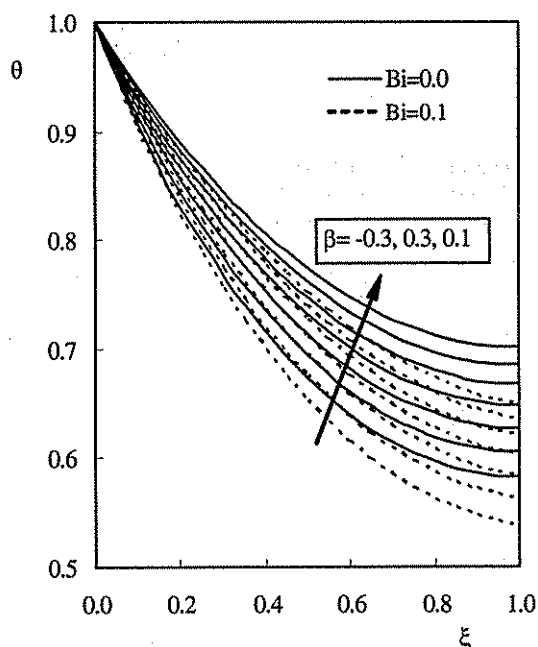
The Adomian decomposition method provides an analytical solution in terms of an infinite power series. The convergence of this series has been established by Cherruault (1989) and Cherruault et al. (1992) for more general nonlinear problems.

In order to investigate the accuracy of the decomposition solution with a finite number of terms, the problem is also solved numerically by using MAPLE which uses a finite difference technique (FDT) with Richardson extrapolation (Ascher and Petzold, 1998), and the corresponding results are compared with the Adomian solution and presented in Table 1. A very good agreement between the results was observed, which confirms the validity of the ADM. From the Table 1, it can also be seen that the results from the decomposition method match with the exact solution which can be taken from the textbook by Incropera and DeWitt (1996) for the case of constant thermal conductivity, i.e., $\beta=0$.

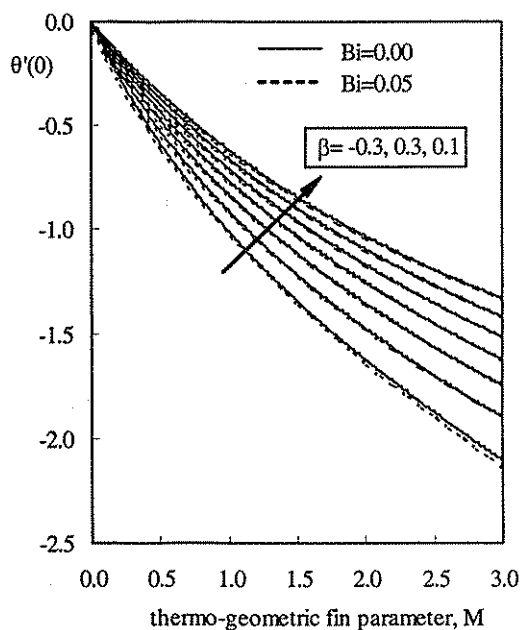
In order to calculate the temperature distribution within the fin for given β , M , and Biot number, it must be evaluated the coefficient C from the boundary condition given in Equation (4c). This calculation is carried out using the Newton-Raphson method. The dimensionless temperature distribution within the fin was calculated by taking the first eight terms from the series solution. The dimensionless temperature distributions along the fin surface with β varying from -0.3 to 0.3 are depicted in Fig.2 for different values of $Bi=0.0$ and $Bi=0.1$. It will be seen that if the thermal conductivity of the fin's material increases with the temperature, i.e. $\beta>0$, the mean temperature increases.

Table 1. Comparison of the present method (ADM) and finite difference technique (FDT) for $M=1.0$, $Bi=0.01$

ξ	Exact	$\beta = 0.0$		$\beta = 0.1$		$\beta = 0.3$	
		ADM	FDT	ADM	FDT	ADM	FDT
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.928300	0.928300	0.928300	0.933334	0.933331	0.941621	0.941493
0.2	0.865891	0.865891	0.865891	0.874852	0.874847	0.889876	0.889616
0.3	0.812148	0.812148	0.812149	0.824145	0.824137	0.844598	0.844203
0.4	0.766534	0.766534	0.766534	0.780852	0.780841	0.805638	0.805101
0.5	0.728591	0.728591	0.728591	0.744660	0.744646	0.772685	0.772171
0.6	0.697940	0.697940	0.697940	0.715302	0.715286	0.746160	0.745295
0.7	0.674274	0.674274	0.674275	0.692561	0.692542	0.725417	0.724372
0.8	0.657357	0.657357	0.657357	0.676264	0.676243	0.710534	0.709322
0.9	0.647019	0.647019	0.647019	0.666287	0.666266	0.701421	0.700086
1.0	0.643156	0.643156	0.643156	0.662556	0.662535	0.698009	0.696627

**Figure 2.** Temperature distribution within the fin.

Figures 3 and 4 show the temperature gradient at the fin base as a function of thermo-geometric fin parameter for different Biot numbers and the thermal conductivity parameters. From the Figures, it can be seen that the effect of the thermo-geometric fin parameter on temperature gradient at the fin base is important. Furthermore, the temperature gradient is strongly depends on the variable thermal conductivity.

**Figure 3** Temperature gradient at the fin base as a function of the thermo-geometric fin parameter for $Bi=0.0$ and $Bi=0.05$.

The dimensionless heat transfer rate is calculated by both ADM and an exact analytical method for constant thermal conductivity. The results are presented in Table 2 for different problem parameters. It has been seen that the results from the ADM perfectly match with the exact solution which can be taken the textbook by Incropera and DeWitt (1996) for the case of constant thermal conductivity.

The dimensionless heat transfer rate varying with M from 0 to 3 are depicted in Figure 5. From the figure, it can be seen that the effects of the thermal conductivity parameter and Biot number on heat transfer rate are important.

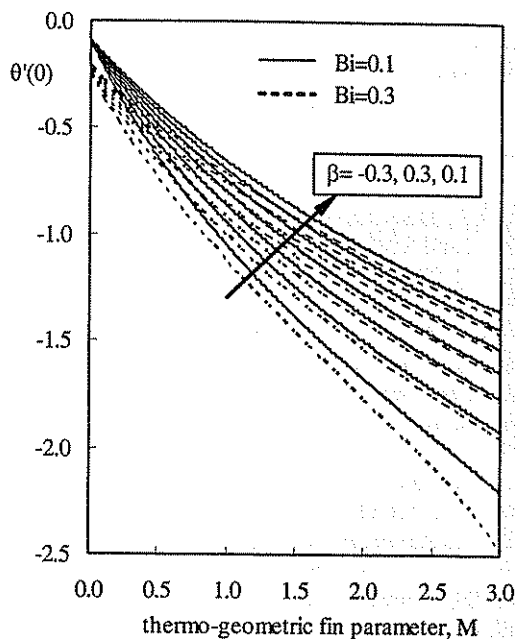


Figure 4 Temperature gradient at the fin base as a function of the thermo-geometric fin parameter for $Bi=0.1$ and $Bi=0.3$.

CONCLUSIONS

Straight fins with convective tip and temperature-dependent thermal conductivity were analyzed using the Adomian decomposition method. The decomposition method supplies reliable results in the form of analytical approximation converging very rapidly. The nonlinear differential equation which governs the fin temperature distribution was solved and

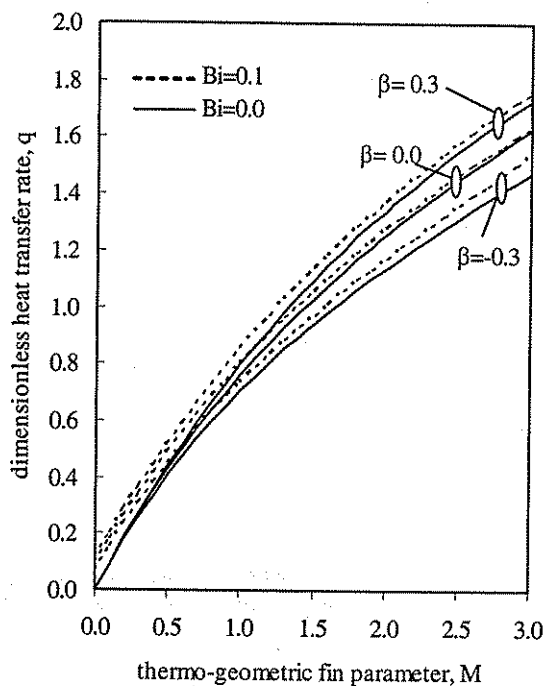


Figure 5 Dimensionless heat transfer rate as a function of the thermo-geometric fin parameter for different Biot numbers and thermal conductivity parameters

then the temperature gradient at the fin base and dimensionless heat transfer rate were calculated. The results were graphically presented in terms of suitable dimensionless parameters. These results can be used for designing straight fins with variable thermal conductivity and convective tip.

Table 2. Dimensionless heat transfer rate for comparison of the present method (ADM) and exact solution for the case of constant thermal conductivity, i.e. $\beta=0$

Bi	M=0.5		M=1.0		M=1.5		M=2.0	
	Exact	ADM	Exact	ADM	Exact	ADM	Exact	ADM
0.00	0.430529	0.430529	0.761594	0.761594	1.030070	1.030070	1.256367	1.256367
0.05	0.460694	0.460694	0.781823	0.781823	1.044216	1.044216	1.266585	1.266585
0.10	0.488469	0.488469	0.800619	0.800619	1.057453	1.057453	1.276198	1.276198
0.15	0.514125	0.514125	0.818132	0.818132	1.069866	1.069866	1.285260	1.285260
0.20	0.537897	0.537897	0.834486	0.834486	1.081529	1.081529	1.293816	1.293816
0.25	0.559984	0.559984	0.849795	0.849795	1.092510	1.092509	1.301908	1.301908
0.30	0.580560	0.580560	0.864154	0.864154	1.102864	1.102864	1.309572	1.309572
0.35	0.599775	0.599775	0.877650	0.877650	1.112646	1.112646	1.316841	1.316841
0.40	0.617759	0.617758	0.890358	0.890358	1.121900	1.121900	1.323745	1.323745
0.45	0.634626	0.634626	0.902345	0.902345	1.130669	1.130669	1.330311	1.330311
0.50	0.650479	0.650479	0.913671	0.913671	1.138990	1.138990	1.336563	1.336563

REFERENCES

- Adomian, G., *Solving frontier problems in physics: The decomposition method*, Kluwer Academic Publishers, 1988a.
- Adomian, G., A review of the decomposition method in applied mathematics, *Journal of Mathematical Analysis and Application*, Vol.135, pp. 501-544. 1988b
- Adomian, G., *Nonlinear stochastic system theory and applications to physics*, Kluwer Academic Publishers, 1994
- Ascher, U., Petzold, L., *Computer methods for ordinary differential equations and differential-algebraic equations*, SIAM, 1998
- Aziz, A., Enamul Hug, S.M., Perturbation solution for convecting fin with variable thermal conductivity, *Journal of Heat Transfer Trans. ASME*, Vol. 97, pp. 300-301, 1975
- Bouaziz, M.N., Recnak, S., Hanini, S., Bal, Y., Bal, K., Étude des transferts de chaleur non linéaires dans les ailettes straightes. *International Journal of Thermal Science*, Vol. 40, pp. 843-857, 2001
- Cherruault, Y., Convergence of Adomian's method, *Kybernets*, Vol. 18, pp. 31-38, 1989
- Cherruault, Y., Saccomandi, G., Some, B., New results for convergence of Adomian's method applied to integral equations, *Mathematical and Computer Modelling*, Vol. 16, pp. 85-93. 1992
- Chiu, C.H., Chen, C.K., A decomposition method for solving the convective longitudinal fins with variable thermal conductivity, *International Journal of Heat and Mass Transfer*, Vol.45, pp. 2067-2075, 2002a
- Chiu, C.H., Chen, C.K., Application of the decomposition method to thermal stresses in isotropic circular fins with temperature-dependent thermal conductivity, *Acta Mechanica*, Vol. 157, pp. 147-158, 2002b
- Incropera, F.P., DeWitt, D.P., *Introduction to heat transfer* (3rd ed.), p.118, Wiley, 1996
- Kern, D.Q., Kraus, D.A., *Extended surface heat transfer*, McGraw Hill, 1972
- Krane, R.J., Discussion on a previously published paper by A.Aziz and S.M. Eug, *Journal of Heat Transfer Trans. ASME*, Vol. 98, pp. 685-686, 1976
- Laor, K., Kalman, H., Performance and optimum dimensions of different cooling fins with a temperature-dependent heat transfer coefficient, *International Journal of Heat and Mass Transfer*, Vol. 39, pp. 1993-2003, 1996
- Lesnic, D., Elliott, L., The decomposition approach to inverse heat conduction, *Journal of Mathematical Analysis and Applications*, Vol. 232, pp. 82-98, 1999
- Lesnic, D., Convergence of Adomian's decomposition method: periodic temperatures, *Computers & Mathematics with Applications*, Vol. 44, pp. 13-24. 2002
- Razelos, P., Imre, K., The optimum dimension of circular fins with variable thermal parameters, *Journal of Heat Transfer Trans. ASME*, Vol. 102, pp. 420-425, 1980
- Sohrabpour, S., Razani, A., Optimum dimensions of convective fin with temperature dependent thermal parameters, *Journal of Franklin Institute*, Vol. 330, 37-49, 1993
- Yu, L.T., Chen, C.K., Application of Taylor transformation to optimize rectangular fins with variable thermal parameters, *Applied Mathematical Modelling*, Vol. 22, pp. 11-21, 1998
- Yu, L.T., Chen, C.K., Optimization of circular fins with variable thermal parameters, *Journal of the Franklin Institute*, Vol. 336B, pp. 75-95, 1999
- Vadasz, P., Olek, S., Transitions and chaos for free convection in a rotating porous layer, *International Journal of Heat and Mass Transfer*, Vol. 41, pp. 1417-1435, 1998.